Fast and Robust Circular Object Detection with Probabilistic Pairwise Voting (PPV)

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***Abstract*—Accurate and efficient detection of circular objects in images is a challenging computer vision problem. Existing circular object detection methods can be broadly classified into two categories: Hough Transform based and maximum likelihood (ML) estimation based. The former is robust to noise, however the computational complexity and memory requirement are high. On the other hand, ML estimation methods (e.g., robust least squares fitting) are more computationally efficient but sensitive to noise, and can not detect multiple circles. This letter proposes Probabilistic Pairwise Voting (PPV), a fast and robust algorithm for circular object detection based on an extension of Hough Transform. The main contributions are three fold: (1) We formulate the problem of circular object detection as finding the intersection of lines in the three dimensional parameter space (i.e. center and radius of the circle). (2) We propose a probabilistic pairwise voting scheme to robustly discover circular objects under occlusion, image noise and moderate shape deformations. (3) We use a mode-finding algorithm to efficiently find multiple circular objects. We demonstrate the benefits of our approach on two real-world problems: i) detecting circular objects in natural images, and ii) localizing iris in face images.**

***Index Terms*—Circular object detection, circular Hough Transform, iris localization.**

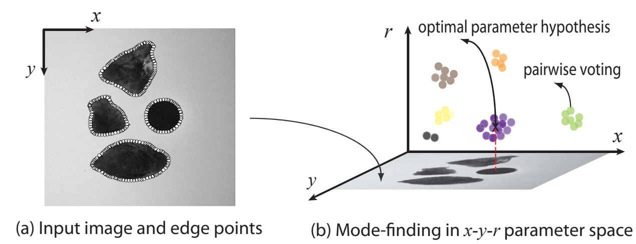

Fig. 1. The proposed circular objection detection method.

I. Introduction

CIRCULAR object detection is an important problem in computer vision due to its wide applicability to problems such as biological cell tracking, inspection (e.g., mechanical parts detection) and biometrics (e.g., iris localization). Existing methods can be broadly classified into two categories: maximum likelihood estimation (MLE) based and voting based. In the following, we will discuss the benefits and drawbacks of both approaches.

The MLE approach to circle fitting was first proposed by Gander *et al.* [1], where they directly estimate the parameter of the circle as a least square estimation problem. In order to improve detection accuracy, Zelniker *et al.* [2] extended [1] with convolution-based MLE to estimate the parameter of circular object. Frosio and Borghese [3] employed prior knowledge of foreground and background statistics to estimate the likelihood of circular objects. Despite substantial improvements, these approaches are still sensitive to noise, partial occlusion and background clutter. Moreover, it is unclear how to extend these approaches to detect multiple circles.

Unlike MLE-based methods, voting-based methods are more robust against noise and occlusion. The most commonly used voting-based algorithm is the Circular Hough Transform (CHT) [4]. A major drawback of the method is the need to build a 3-dimensional accumulator array of circle parameter (i.e. center and radius), resulting in high computational complexity and memory requirements. In [5], Xu *et al.* reduced computational complexity by randomly selecting a subset of edge points for voting. Another approach for reducing the computational expense of CHT is to leverage gradient information of every edge point. Valenti and Gevers [6] utilized the first and the second order gradient derivatives to estimate the curvature and radius of every possible circular object. In practice, estimating the radius based on gradient derivatives can be inaccurate in the presence of noise and shape deformations. The work of [7] and [8] used 2-D gradient lines and calculated their intersections for circular object detection. These two methods improved the efficiency of circular object detection substantially. However, they suffer from two inherent drawbacks: firstly, it is unclear how to estimate the radius of circular objects from 2-D gradient lines in image space only; secondly, iterative search is required when detecting multiple circular objects.

To circumvent the aforementioned limitations in existing methods, this work proposes a fast and robust circular object detection method based on three key ideas. First, we parameterize the circles as lines in the three dimensional parameter space. This allows one to estimate the radius and center rapidly together. Second, we use a probabilistic weighting scheme to improve robustness of the detection. Third, we use a fast mode-finding algorithm that allows the rapid and robust detection of multiple circular objects. Fig. 1 illustrates the idea of the letter.

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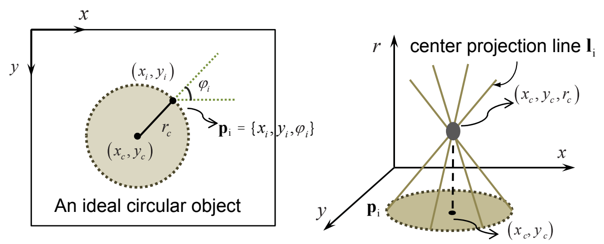


(a) An ideal circular object (b) Center projection lines

Fig. 2. Center projection line analysis.

II. The Proposed Algorithm

A. Center Projection Line

Consider the edge map of a noiseless image with a single ideal circular object, as shown in Fig. 2(a). The center location and radius of this circular object is represented by $(x_c, y_c)^T \in \mathbb{R}^2$ and $r_c \in \mathbb{R}$, respectively. For each edge point, we have $\mathbf{p}_i = \{x_i, y_i, \varphi_i\}$, where $(x_i, y_i)^T \in \mathbb{R}^2$ is the pixel location and $\varphi_i \in \mathbb{R}$ is the angle of the gradient direction. Based on the fact that the gradient direction of an edge point of circular object points either towards or away from its center, we have the following relationships:

$$x_c = x_i - r_c \cos\varphi_i \quad \text{and} \quad y_c = y_i - r_c \sin\varphi_i. \tag{1}$$

Here, we consider the case where the edge gradient direction is pointing away from the object's center. Once the circle is parameterized by Eq. (1), it is easy to see that the parameter of all possible circular objects that contain $\mathbf{p}_i$ lie on a 3-D line $\mathbf{l}_i$ in x-y-r parameter space. We will refer to this line as the center projection line and illustrate it in Fig. 2(b). The parametric equation of the line $\mathbf{l}_i$ takes the form:

$$\begin{pmatrix} x \\ y \\ r \end{pmatrix} = \begin{pmatrix} x_i \\ y_i \\ 0 \end{pmatrix} + r \begin{pmatrix} -\cos\varphi_i \\ -\sin\varphi_i \\ 1 \end{pmatrix}, \; r > 0. \tag{2}$$

Fixing the radius, $r = r_c$, implies that the object's center location is (x_c, y_c). For a noiseless image with a single ideal circular object, all edge points are on the object's boundary and their center projection lines intersect at a single point $\mathbf{x}_c = (x_c, y_c, r_c)^T \in \mathbb{R}^3$ in the parameter space (see Fig. 2(b)). This intersection corresponds to the circular object's parameter.

In practice, image noise and background clutter often result in multiple intersection points. Here, the optimal point $\mathbf{x}_c$ is defined as the point through which the maximum number of center projection lines pass. This can be found through pairwise voting: calculating the intersection point of every pair of center projection lines and voting for the point with maximum intersections. Unlike previous approaches to circle detection based on the 2-D gradient lines [7], [8], our approach can obtain the center position and radius simultaneously by virtue of formulating center projection lines in the 3-D parameter space. This is one of our main contributions.

B. Probabilistic Pairwise Voting

Due to noise and deviations from an ideal circle, center projection lines may not intersect in the parameter space even when they are associated with the same circular object. Thus, the hard voting schemes used in standard Hough Transform are not adequate. Instead, we propose a pairwise voting procedure under a probabilistic framework that is robust against small shape deformations and noise. This formulation also enables the search for the optimal parameter hypothesis in continuous space through the use of mode-finding algorithms.

Given two edge points $\mathbf{p}_i$ and $\mathbf{p}_j$, we first calculate the probability that they are located on the same circular object, denoted by $p\left(o_{ij} = \mathcal{O} | \mathbf{p}_i, \mathbf{p}_j\right)$. Here, o_{ij} is a random variable representing whether $\mathbf{p}_i$ and $\mathbf{p}_j$ are both on the same circular object (i.e. satisfying hypothesis '$\mathcal{O}$') or not. If they are on the same circular object boundary, their corresponding center projection lines should intersect in the ideal case. Although the existence of shape deformations and noise in real-world images means that this is often not the case, the minimum distance (after scale normalization) between these lines is a reasonable measure to use as it is related to the degree to which the object is deformed, and thus, how *circle-like* it is.

The minimum distance between two center projection lines can be found as follows. First, observe that any two lines, $\mathbf{l}_i$ and $\mathbf{l}_j$, are intersected by a third that is perpendicular to both (see Fig. 3). Let us denote by $\mathbf{x}_{ij} = (x_{ij}, y_{ij}, r_{ij})^T$ and $\mathbf{x}_{ji} = (x_{ji}, y_{ji}, r_{ji})^T$ the intersection points of the perpendicular line with the other two. $\mathbf{x}_{ij}$ and $\mathbf{x}_{ji}$ are calculated based on the truth that the inner product of two perpendicular vectors is zero.$\|\mathbf{x}_{ij} - \mathbf{x}_{ji}\|_2$ is the minimum distance between the two center projection lines. Then, we model the likelihood that two edge points correspond to the same circular object as:

$$\pi_{ij} = \begin{cases} \frac{1}{C} \exp\left(\frac{-\|\mathbf{x}_{ij} - \mathbf{x}_{ji}\|_2^2}{\bar{r}_{ij}^2 t} \right) & \text{if } \frac{\|\mathbf{x}_{ij} - \mathbf{x}_{ji}\|_2}{\bar{r}_{ij}} < \tau \\ 0 & \text{otherwise} \end{cases}, \tag{3}$$

where π_{ij} is shorthand for $p\left(o_{ij} = \mathcal{O} | \mathbf{p}_i, \mathbf{p}_j\right)$, $\bar{r}_{ij} = \frac{(r_{ij} + r_{ji})}{2}$ is a normalization factor that eliminates the effects of size variations, and C is a normalizing constant that ensures π_{ij} is a proper probability. t and τ are positive constants that trade off detection accuracy against shape deformation robustness. From Eq. (3), we can see this likelihood term is equal to zero if the minimum normalized distance between two center projection lines is larger than τ. This follows from the intuition that the probability of two edge points belonging to the same circular object is zero if this minimum distance is large.

Each pair of edge points (including their locations and gradient directions) lines to the distribution of circular object parameters. Specifically, the likelihood of the circular object's parameter $\theta = (x_c, y_c, r_c)^T$ given two edge points lying on it is modeled as an isotropic Gaussian:

$$p\left(\theta | o_{ij} = \mathcal{O}, \mathbf{p}_i, \mathbf{p}_j\right) = \mathcal{N}\left(\theta; \bar{\mathbf{x}}_{ij}, \sigma_{ij}^2 \mathbf{I}\right), \tag{4}$$

where $\bar{\mathbf{x}}_{ij} = \frac{\mathbf{x}_{ij} + \mathbf{x}_{ji}}{2}$ is the point that is closest to both projection lines $\mathbf{l}_i$ and $\mathbf{l}_j$ (see Fig. 3). In Eq. (4), $\sigma_{ij} = \bar{r}_{ij} w$, where w is a constant scaling factor. Combining Eqs. (3) and (4) and using Bayes' theorem, then the conditional probability of both edge points lying on the boundary of the same circular object whose center and radius are represented by θ is:

$$p\left(o_{ij} = \mathcal{O}, \theta | \mathbf{p}_i, \mathbf{p}_j\right) = p\left(\theta | o_{ij} = \mathcal{O}, \mathbf{p}_i, \mathbf{p}_j\right) p\left(o_{ij} = \mathcal{O} | \mathbf{p}_i, \mathbf{p}_j\right), \tag{5}$$

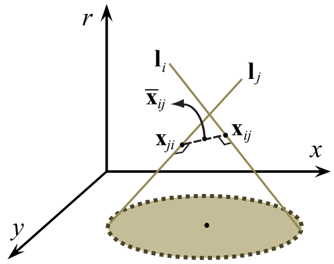


Fig. 3. Pairwise probabilistic voting analysis.

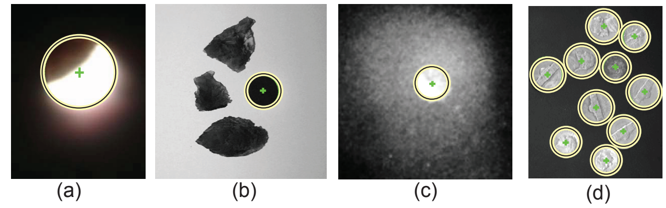


Fig. 4. Circle detection results. (a) occlusion; (b) background clutter; (c) shape deformation; (d) multiple circular objects.

C. Optimal Hypothesis Search

The score of a parameter hypothesis can be found by marginalizing over all edge point pairs that contribute to this hypothesis:

$$S(\mathcal{O}, \theta) = \sum_i \sum_j p\left(o_{ij} = \mathcal{O}, \theta | \mathbf{p}_i, \mathbf{p}_j\right) p(\mathbf{p}_i, \mathbf{p}_j). \quad (6)$$

Assuming $p(\mathbf{p}_i, \mathbf{p}_j)$ is a uniform distribution:

$$\begin{aligned} S(\mathcal{O}, \theta) &\propto \sum_i \sum_j p\left(o_{ij} = \mathcal{O}, \theta | \mathbf{p}_i, \mathbf{p}_j\right) \\ &\propto \sum_i \sum_j \pi_{ij} \; \mathcal{N}\left(\theta; \bar{\mathbf{x}}_{ij}, \sigma_{ij}^2 \mathbf{I}\right). \end{aligned} \quad (7)$$

The optimal hypothesis corresponds to the maximum of this distribution. Following [9], we can use quadratic maximization combined with gradient ascent to find the local modes of the underlying mixture of Gaussian distributions rapidly or just use mean shift. For multiple circular objects detection, we simply retain the k-best modes. k is the number of modes whose corresponding score (defined in Eq. (6)) normalized by scale is larger than a threshold ζ.

The computational complexity of our algorithm is $O\left(N^2 + hM\right)$, where N is the number of edge pixels, M the number of non-zero components in Eq. (7), and h the number of steps that the mode-finding algorithm takes to converge. The memory requirement of our algorithm is $O(M)$. In comparison, for CHT [4], the computational cost is $O(n^2 N)$ and the memory requirement is $O(n^3)$ given the size of parameter space is $n \times n \times n$. Because the number of iterations h is typically small, and $N \ll n^2$, the computational complexity and memory requirements of the proposed algorithm are lower than CHT. Moreover, we can speed up our method by randomly sampling edge points.

III. Experiments

We demonstrate the effectiveness of the proposed algorithm on two tasks: detecting circular objects in natural images and localizing iris in face images. All experiments were implemented in Matlab on 2.80 GHz Intel Core 2 Duo processor.

A. Detecting Circular Objects in Natural Images

We compare PPV against a number of existing methods on four typical natural images gathered from Google Image[1]. The selected four representative images included partial occlusion, background clutter, shape deformation and multiple circular objects (see Fig. 4). The sizes of the four test images are 400×286, 429×440, 224×225 and 406×356 respectively. We compared PPV with three circle detection algorithms: 1) Circular Hough Transform (CHT) [4], 2) Randomized Circle Detection (RCD) [10], and 3) Approximate Maximum-Likelihood Estimation (AMLE) [2]. Each input image was smoothed with a Gaussian filter with standard deviation 2.0, and an edge map was then generated for each image using a Canny edge detector with a low (high) threshold of 0.16 (0.4). For PPV, we set the threshold τ to be 0.2 and fixed $t = 0.2\tau$ in Eq. (4). For CHT, we used the *hard* voting scheme.

[1]Find at http://humansensing.cs.cmu.edu/wschu/project_circdet.html

TABLE I
DETECTION RESULT ANALYSIS ON NATURAL SCENE IMAGES

algorithm	Fig.4 (a)	Fig.4 (b)	Fig.4 (c)	Fig.4 (d)
CHT [4]				
error in (x_c, y_c)	0.5	1.4	8.6	1.0
error in r_c	2.0	1.0	6.0	0.4
time	79.32s	179.70s	8.61s	176.88s
RCD [10]				
error in (x_c, y_c)	1.5	1.4	0.6	0.5
error in r_c	1.0	1.0	0.0	0.5
time	2.08s	4.16s	1.91s	5.12s
AMLE [2]				
error in (x_c, y_c)	2.2	142.2	0.3	-
error in r_c	2.5	10.3	0.5	-
time	0.08s	0.12s	0.03s	-
PPV				
error in (x_c, y_c)	3.2	0.8	0.4	0.6
error in r_c	0.0	0.5	0.1	0.7
time	0.59s	0.86s	0.13s	3.18s

The detection results of PPV are shown in Fig. 4. For each image, the performance of the different methods were evaluated using the Root Mean Square (RMS) error of the center position (x_c, y_c) and radius r_c. The ground-truth was manually labeled. The error and the running time for each algorithm is reported in Table I.

The proposed algorithm has 98.93% and 40.08% execution time improvement over CHT and RCD respectively. Although AMLE was faster than PPV, it was more sensitive to background clutter and noise. This can be seen in the second column of Table I, where AMLE has the largest RMS-error when tested on Fig. 4(b). Moreover, MLE-based approaches can not account for multiple circular objects.CHT has relatively large RMS-error on Fig. 4(c) since it is based on hard voting and thus can not tolerate shape deformations. The RCD method has comparable accuracy to PPV, but it is slower.

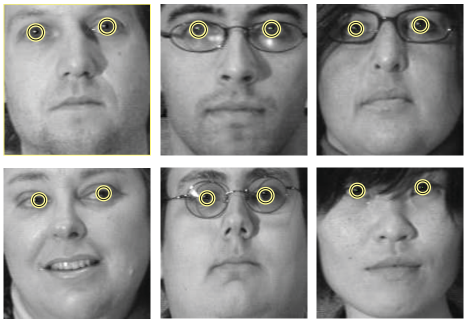
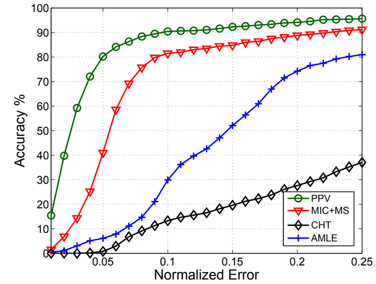


Fig. 5. Eye localization results on CMU Multi-PIE database.

τ	$e < 0.05$	$e < 0.1$
0.1	77.83%	86.09%
0.2	79.13%	89.67%
0.3	79.46%	90.22%
0.4	80.46%	90.43%
0.5	79.35%	90.33%
0.6	78.91%	89.33%

(a)



(b)

Fig. 6. Our results: (a) localization accuracy under different τ and normalized error e, (b) cumulative error distribution curves.

B. Localizing the Iris in Face Images

To further demonstrate PPV's effectiveness, we tested it on the challenging task of localizing the iris in face images. We used 920 frontal face images from 337 persons from the CMU Multi-PIE face database [11]. Finding iris in face image is a particularly challenging problem because of reflections from eye glasses, occlusions from eyelash, eyelid and hair. Moreover, out-of-axis gazing is also present in some images.

To evaluate the eye localization accuracy, we used the *normalized error* as in [6]:

$$e = \frac{\max(d_l, d_r)}{\|C_l - C_r\|}, \tag{8}$$

where d_l is the Euclidean distance between the localized left eye center and the ground-truth. Similarly, d_r is defined for the right eye. C_l and C_r are the ground truth positions for left and right eye centers.

We first evaluated the effect of the parameter τ in Eq. (3) on the localization performance. This parameter trades off robustness to shape deformation against localization accuracy. Through out this experiment, the low and high thresholds for the Canny operation were set at 0.1 and 0.25. All local modes in parameter space were obtained. For each mode, we determined which edge point pairs contribute to this hypothesis and calculate the sum of their gradient derivatives. Finally, the two consistent hypotheses whose related edge points' gradient derivative sums are maximal were selected. Since some subjects wear circle-like shape glasses, we imposed scale constraints for the final selection. We used $e < 0.05$ and $e < 0.1$ to measure the localization performance of our algorithm with different τ values ($t = 0.2\tau$), and the localization accuracy are reported in Fig. 6(a). The best performance was obtained for $\tau = 0.4$.

Apart form CHT and AMLE, we also compared PPV with the state-of-art circle detection based iris localization algorithms: Maximum Isocenter + Mean Shift (MIC+MS) [6]. In every baseline method, we detected the two circles whose gradient magnitude integration along circular object edge are maximal. For better comparison, the localization was applied in the upper face region (including two eyes, eyebrows, hair, eye glasses etc.) instead of detected eye regions used in [6]. Fig. 6(b) plots the accuracy of the four methods for different normalized errors. Larger area under the curve indicates a better performance. Because MIC estimates radii by gradient derivative, it is inaccurate under noise and small shape deformations in real-world. On the other hand, PPV considers shape deformations and noise when calculating every vote and hence can achieve better results. Similarly, the *hard* voting scheme used in the traditional CHT fails to account for non-circular shape deformations, leading to poor performance. Moreover, because AMLE is sensitive to noise and background cluster, it also have inferior performance to PPV. The average computational costs for CHT, MIC+MS, AMLE and PPV are 23.77s, 0.11s, 0.12s and 0.30s.

IV. Conclusion

This work proposes PPV for fast and robust circular object detection. PPV is robust to occlusion, noise, and small shape deformations and it can detect multiple circular objects in a single image. Experiments of detecting circular objects in natural images and localizing iris in face images illustrates the benefits of PPV against the state-of-the-art methods.

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